

Exam 3: Ch 5 & 6

Math 176, Precalculus, Section 6265

Spring 2007: Michael Orr

NAME Answer Key  
 NO TI-89 CALCULATORS!!!!!!!

**100 points. Show all work to receive full credit. You may use a calculator. CHECK YOUR WORK!!!!**

1. (3 pts each) Find the EXACT value of each trig function. (NOTE: decimals are NOT exact!)

a.  $\cos(-270^\circ)$

$\boxed{0}$

b.  $\sin \frac{7\pi}{6}$

$= \boxed{-\frac{1}{2}}$

c.  $\sec\left(-\frac{2\pi}{3}\right)$

$= \boxed{-2}$

d.  $\tan(3\pi)$

$\boxed{0}$

e.  $\csc(135^\circ)$

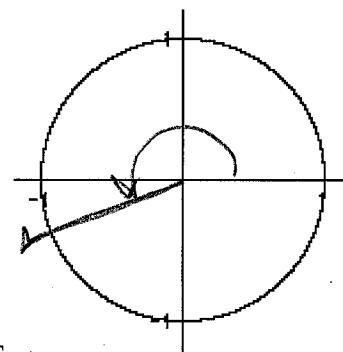
$= \frac{2\sqrt{2}}{2} = \boxed{\sqrt{2}}$

f.  $\cot 300^\circ = \frac{\cos 300}{\sin 300}$

$= \frac{+\frac{1}{2}}{-\frac{\sqrt{3}}{2}} = \boxed{-\frac{\sqrt{3}}{3}}$

2. (9 pts) Consider the angle  $\theta = \frac{10\pi}{9}$  radians.

a. Draw (approximately) the angle  $\theta$  in standard position.



②

b. Convert  $\theta = \frac{10\pi}{9}$  into degrees. Your answer should be EXACT.

③

$\frac{10\pi}{9} \times \frac{180}{\pi} = \boxed{200^\circ}$

c. Using your calculator, determine the terminal point  $P(x,y)$  on the unit circle associated with

$\theta = \frac{10\pi}{9}$ .  $x$  and  $y$  should be correct to 4 decimals.

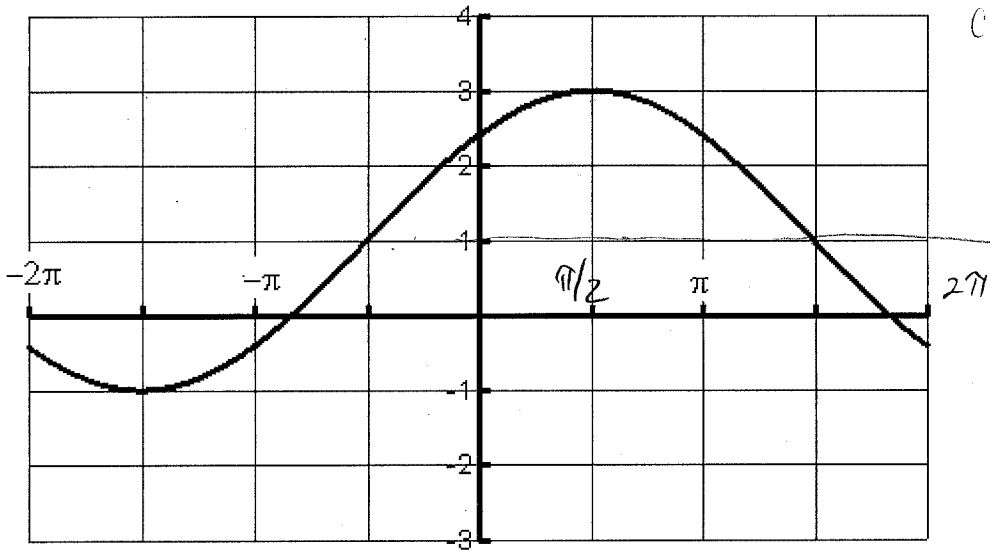
④

$x = \cos 200^\circ = \cos \frac{10\pi}{9}$

$y = \sin 200^\circ = \sin \frac{10\pi}{9}$

$\boxed{(-0.9397, -0.3420)}$

3. (6 pts) The graph of a cosine function of the form  $y = a \cos k(x-b) + c$  is graphed below. Determine the function.

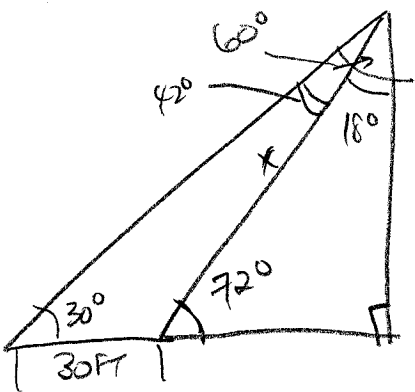


$$\begin{aligned} \frac{1}{2}(x - \frac{\pi}{2}) &= 0 \\ \frac{1}{2}x - \frac{\pi}{4} &= 0 \\ \frac{1}{2}x &= \frac{\pi}{4} \\ x &= \frac{\pi}{2} \end{aligned}$$

$a = 2$   
 $c = +1$   
 $T = \frac{2\pi}{k} = 4\pi$   
 $k = \frac{1}{2}$   
 $b = \frac{\pi}{2}$

$y = 2 \cos \frac{1}{2} \left( x - \frac{\pi}{2} \right) + 1$

4. (6 pts) A guy wire to a tower makes a  $72^\circ$  angle with level ground. At a point 30 ft farther from the tower than the wire but on the same side as the wire, the angle of elevation to the top of the tower is  $30^\circ$ . Find the length of the wire (to the nearest foot).



$$\frac{\sin 30}{x} = \frac{\sin 42}{30}$$

$$x = \frac{30 \sin 30^\circ}{\sin 42^\circ}$$

$$x = 22.4$$

$x = 22 \text{ ft}$

5. Consider the functions  $y = 2 \cos(4x)$  and  $y = 2 \sec(4x)$ .

a. (2 pts) Find the amplitude of  $y = 2 \cos(4x)$ .

AMPLITUDE = 2

b. (4 pts) Find the period of each function.

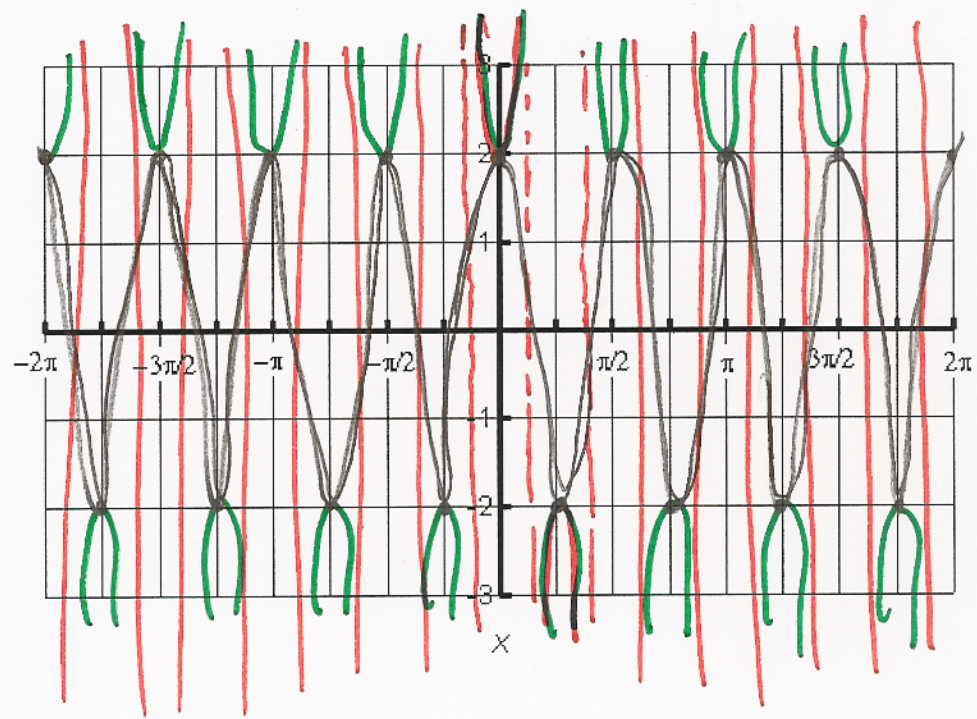
$$T = \frac{2\pi}{k} = \frac{2\pi}{4} = \frac{\pi}{2}$$

Period of  $y = 2 \cos(4x)$ :  $\frac{\pi}{2}$

$$T = \frac{2\pi}{k} = \frac{2\pi}{4} =$$

Period of  $y = 2 \sec(4x)$ :  $\frac{\pi}{2}$

c. (6 pts) On the grid below, sketch the graph of  $y = 2 \cos(4x)$ .

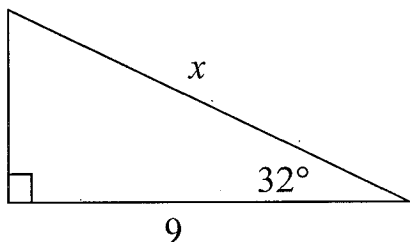


d. (6 pts) On the same set of axes, sketch the graph of  $y = 2 \sec(4x)$ . Show any asymptotes.

ASYMPTOTES @  $\pm \frac{\pi}{8} \pm n \frac{\pi}{2}$

(5 pts each) For problems 5 – 9, solve for either  $x$  or  $\theta$  in the following triangles. Round to 2 decimals.

5.

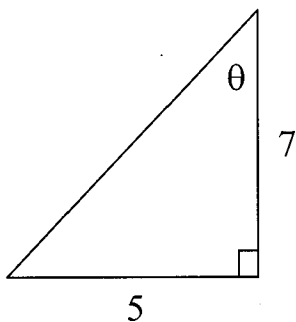


$$\cos 32^\circ = \frac{9}{x}$$

$$x = \frac{9}{\cos 32^\circ}$$

$$= \boxed{10.61}$$

6.

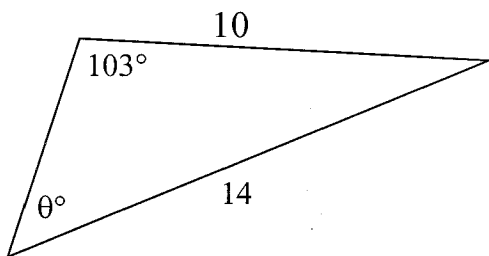


$$\tan \theta = \frac{5}{7}$$

$$\theta = \tan^{-1}\left(\frac{5}{7}\right)$$

$$\theta = \boxed{35.54^\circ}$$

7.

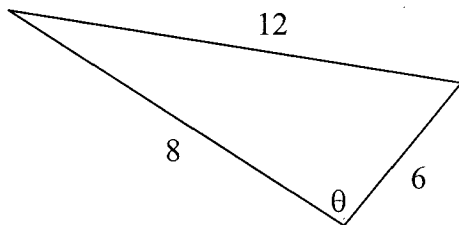


$$\frac{\sin 103}{14} = \frac{\sin \theta}{10}$$

$$\sin \theta = \frac{10}{14} \sin 103$$

$$\theta = \sin^{-1}\left(\frac{10}{14} \sin 103\right) = \boxed{44.11^\circ}$$

8.



$$12^2 = 8^2 + 6^2 - 2(8)(6) \cos \theta$$

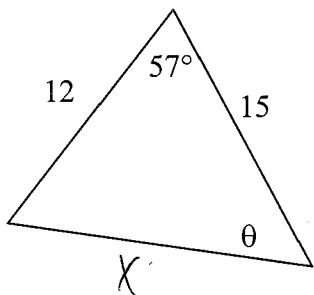
$$144 = 64 + 36 - 96 \cos \theta$$

$$144 = 100 - 96 \cos \theta$$

$$\frac{44}{-96} = \cos \theta \quad \theta = \cos^{-1}\left(\frac{-44}{96}\right)$$

$$\theta = \boxed{117.28^\circ}$$

9.



$$\frac{\sin 57}{x} = \frac{\sin \theta}{12}$$

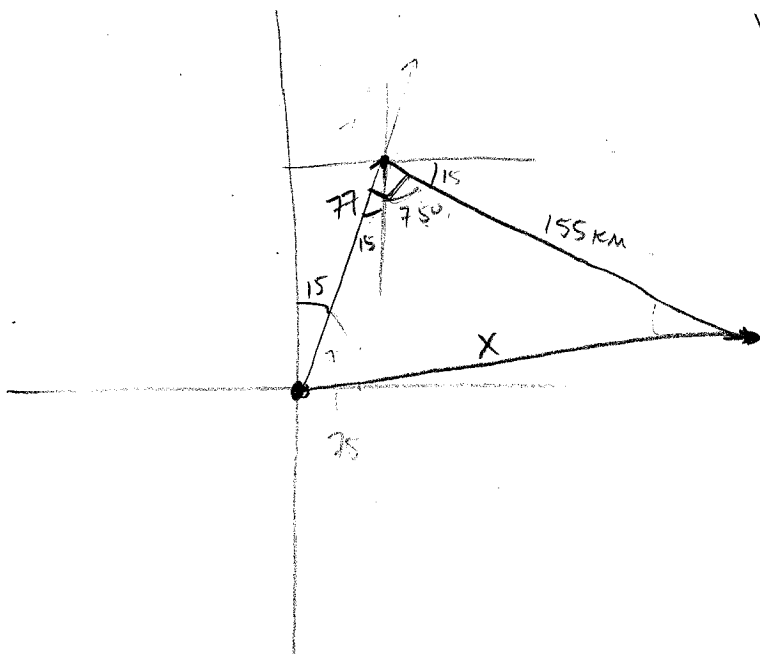
$$\theta = \sin^{-1}\left(\frac{12 \sin 57^\circ}{x}\right)$$

$$x^2 = 12^2 + 15^2 - 2(12)(15) \cos 57^\circ$$

$$x = 13.15$$

$$\theta = \boxed{49.9^\circ \quad (49.7^\circ)}$$

10. (8 pts) A ship travels 77 km on a bearing of  $N15^\circ E$ , and then travels on a bearing of  $S75^\circ E$  for 155 km. Find the distance of the end of the trip from the starting point, to the nearest kilometer.



$$X^2 = 155^2 + 77^2 - 2(155)(77) \cos 90^\circ$$

$$X = \sqrt{155^2 + 77^2}$$

$$X = \sqrt{29954}$$

$$X = 173 \text{ km}$$

11. (2 pts each) True or False. Determine whether each statement is True or False

- a. If  $\sin \theta = \frac{4}{9}$  and  $\theta$  is in quadrant II, then  $\tan \theta = \frac{4}{\sqrt{65}}$ . FALSE
- b.  $\csc \theta$  is positive when  $\theta$  is in quadrant IV. FALSE
- c. The Law of Sines is only true for right triangles. FALSE
- d.  $-\frac{\pi}{4}, \frac{7\pi}{4}$ , and  $\frac{15\pi}{4}$  are all coterminal angles. TRUE
- e.  $\sin^2 t = \sin(t^2)$  for all real numbers  $t$ . FALSE



**BONUS** (total of 10 extra points)



- A. A satellite 200 miles above the earth is orbiting the earth once every 6 hours. How far does the space shuttle travel in 1 hour? (Assume the radius of the earth is 4,000 miles.) Round your answer to the nearest mile.

① No solution - SPACE SHUTTLE NOT SATELLITE,

- B. In an automobile transmission, a gear ratio  $g$  is the ratio  $g = \frac{\text{angular speed of engine}}{\text{angular speed of wheels}}$ .

The angular speed of the engine is shown on the tachometer (in rpm).

A certain sports car has wheels with radius of 11 inches. Its gear ratios are shown in the following table. Suppose the car is in fourth gear and the tachometer reads 2500 rpm.

Gear	Ratio
1 <sup>st</sup>	4.1
2 <sup>nd</sup>	3.0
3 <sup>rd</sup>	1.6
4 <sup>th</sup>	0.9
5 <sup>th</sup>	0.7



- ③ Find the angular speed of the engine.  
 ③ Find the angular speed of the wheels.  
 ③ How fast (in MPH) is the car traveling?

A.)  $2500 \text{ RPM} \times \frac{2\pi \text{ RAD}}{\text{REV}}$   
 $= 15707.96 \frac{\text{RAD}}{\text{MIN}}$

B.)  $\omega = \frac{15707.96 \text{ RAD/MIN}}{0.9}$

$\omega = 17453.29 \frac{\text{RAD}}{\text{MIN}}$

C.)  $v = r\omega = 17453.29 \frac{\text{RAD}}{\text{MIN}} \times \frac{60 \text{ MIN}}{\text{HR}} \times 11 \text{ IN} \times \frac{1 \text{ FT}}{12 \text{ IN}} \times \frac{1 \text{ M}}{1.5280}$

$v = 181.8 \frac{\text{MILES}}{\text{HR}}$